

Math 10A

Worksheet, Midterm II Review; Thursday, 7/19/2018

Instructor name: Roy Zhao

1 FTC

1. True False $\int_0^x e^{t^2} dt$ is an antiderivative of e^{x^2} .
2. If $\int_0^x f(t)dt = \frac{1}{2} \cos(2x) - a$, find f, a .
3. Find the derivative of $\int_{\pi}^x \sec(t)dt$.
4. Find the derivative of $\int_{\pi}^x \sec(t)dt$.
5. Find the derivative of $\int_x^3 e^{-t^2} dt$.
6. Find the derivative of $\int_0^{x^3} \ln(t)dt$.
7. Find the derivative of $\int_{2x}^{x^2} \sqrt{t^2 + t} dt$.

2 U-Substitution/Integration by Parts

8. True False When integrating by parts, choosing different functions for u and dv (assuming both work out), will give different answers.
9. True False It is always good to u sub first in order to simplify the integral.
10. Integrate $\int x(3x^2 - 5)^5 dx$.
11. Integrate $\int 2x^3 e^{x^2} dx$.
12. Find $\int_0^1 \sqrt{1 - \sqrt{x}} dx$.
13. Find $\int x^5 e^{x^3} dx$.
14. Integrate $\int 2x^3 \cos(x^2) dx$.
15. Integrate $\int 2x \arctan(x) dx$.

16. Integrate $\int \frac{\ln \sqrt{x}}{\sqrt{x}} dx$.
17. Integrate $\int_0^{\pi/2} \sin(x) \cos(x) \sin(\sin(x)) dx$.
18. Integrate $\int_0^1 2x^3 \sin(x^2) dx$.
19. Integrate $\int_0^1 x^{-1/2} \arctan(\sqrt{x}) dx$.
20. Integrate $\int_1^{e^\pi} \sin(\ln(x)) dx$.

3 Symmetry

21. Is $f(x) = \frac{x \sin(x)}{x^2 + 4}$ even, odd, or neither?
22. Is $f(x) = x^2 \tan(x)$ even, odd, or neither?
23. Is $f(x) = xe^x$ even, odd, or neither?
24. Is $f(x) = e^{x^2} \sin(x)$ even, odd, or neither?

4 Numerical Integration

25. True False Numerical approximations are just approximations, and never the exact answer.
26. True False The second derivative can tell us if the midpoint rule gives an over/under estimate.
27. True False Simpson's method will approximate cubics exactly.
28. True False When calculating K_1 of $f(x)$ on $[a, b]$, we have that K_1 is the maximum of $|f'(a)|$ and $|f'(b)|$.
29. How many intervals do we need to use to approximate $\int_1^4 \ln x dx$ within $0.001 = 10^{-3}$ using Simpson's rule? Approximate it using Simpson's rule and $n = 4$.
30. How many intervals do we need to use to approximate $\int_{-3}^{-1} 1/x^2 dx$ within $0.001 = 10^{-3}$ using the midpoint rule? Approximate it using the midpoint rule and $n = 4$.
31. How many intervals do we need to use to approximate $\int_0^4 e^x dx$ within $0.001 = 10^{-3}$ using the trapezoid rule? Approximate it using the trapezoid rule and $n = 4$.

32. Approximate the integral $\int_1^3 \frac{dx}{x}$ with $n = 2$ intervals using the different methods (left endpoint, right endpoint, midpoint, trapezoid, Simpson's).
33. What is the smallest value of n needed to ensure that our numerical approximation method for $\int_1^3 dx/x$ is within $0.0001 = 10^{-4}$ using the different methods?

5 Improper Integrals

34. True False We can compare an integral to $\int_1^\infty 1/\sqrt{x}dx$ in order to show it converges.
35. True False We can compare an integral to $\int_1^\infty 1/x^2dx$ to show it diverges.
36. True False Since $x < x + 1$, we have that $\infty = \int_1^\infty \frac{1}{x}dx \leq \int_1^\infty \frac{1}{x+1}dx$ so the latter integral diverges.
37. Calculate $\int_{-\infty}^\infty \frac{1}{1 + (x - 1)^3}dx$.
38. Calculate $\int_1^\infty xe^{-2x}dx$.
39. Calculate $\int_1^\infty \frac{2x}{1 + x^2}dx$.
40. Does $\int_3^\infty \frac{1}{\sqrt{x} \ln(x)}$ converge?
41. Does $\int_1^\infty \frac{2x + 2xe^{-x}}{1 + x^2}dx$ converge?

6 Partial Fractions

42. Integrate $\int \frac{5x+17}{x^2+2x-15}dx$.
43. Integrate $\int \frac{2x^3-12x^2+28x-23}{(x-2)^2(x-1)^2}dx$.
44. Set up the partial fraction decomposition of $\frac{3x^2+1}{(x-1)(x^2+4)^2(x^2+2x+2)^2}$ (you don't have to solve for the coefficients).